Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2001 Solutions

Gauss Contest

(Grades 7 and 8)

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Part A

1. The largest number in the set $\{0.01, 0.2, 0.03, 0.02, 0.1\}$ is

(**A**) 0.01 (**B**) 0.2 (**C**) 0.03 (**D**) 0.02 (**E**) 0.1

Solution

If we write each of these numbers to two decimal places by adding a '0' in the appropriate hundredths column, the numbers would be 0.01, 0.20, 0.03, 0.02 and 0.10. The largest is 0.2.

ANSWER: (B)

2. In 1998, the population of Canada was 30.3 million. Which number is the same as 30.3 million?

(**A**) 30 300 000 (**B**) 303 000 000 (**C**) 30 300 (**D**) 303 000 (**E**) 30 300 000 000

Solution

In order to find what number best represents 30.3 million, it is necessary to multiply 30.3 by 1 000 000. This gives the number 30 300 000.

ANSWER: (A)

3. The value of $0.001+1.01+0.11$ is

(**A**) 1.111 (**B**) 1.101 (**C**) 1.013 (**D**) 0.113 (**E**) 1.121

Solution

If we add these numbers 0.001, 1.01 and 0.11, we get the sum 1.121. We can most easily do this by calculator or by adding them in column form,

$$\begin{array}{r} 0.001 \\ 1.01 \\ +0.11 \\ \hline 1.121 \end{array}$$

ANSWER: (E)

4. When the number 16 is doubled and the answer is then halved, the result is

(**A**) 2^1 (**B**) 2^2 (**C**) 2^3 (**D**) 2^4 (**E**) 2^8

Solution

When the number 16 is doubled the result is 32.

When this answer is halved we get back to 16, our starting point. Since $16=2^4$, the correct answer is 2^4.

ANSWER: (D)

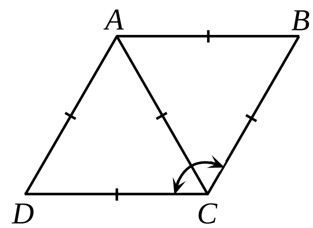
5. The value of $3\times 4^2-(8\div 2)$ is

(**A**) 44 (**B**) 12 (**C**) 20 (**D**) 8 (**E**) 140

Solution
Evaluating, $3 \times 4^2 - (8 \div 2)$
$= 48 - 4$
$= 44.$

ANSWER: (A)

6. In the diagram, $ABCD$ is a rhombus. The size of $\angle BCD$ is
(**A**) 60° (**B**) 90° (**C**) 120°
(**D**) 45° (**E**) 160°



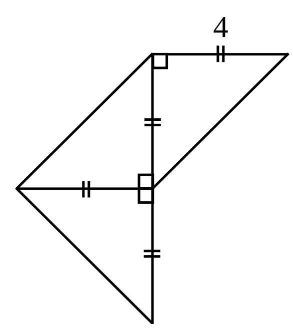
Solution
We are given that ΔADC is equilateral which means that $\angle DAC = \angle ACD = \angle ADC = 60°$. Similarly, each of the angles in ΔABC equals 60°. This implies that $\angle BCD$ equals 120° since $\angle BCD = \angle BCA + \angle DCA$ and $\angle BCA = \angle DCA = 60°$. ANSWER: (C)

7. A number line has 40 consecutive integers marked on it. If the smallest of these integers is –11, what is the largest?
(**A**) 29 (**B**) 30 (**C**) 28 (**D**) 51 (**E**) 50

Solution
We note, first of all, that 0 is an integer. This means from –11 to 0, including 0, that there are 12 integers. The remaining 28 mark from 1 to 28 on the number line. The largest integer is 28.

ANSWER: (C)

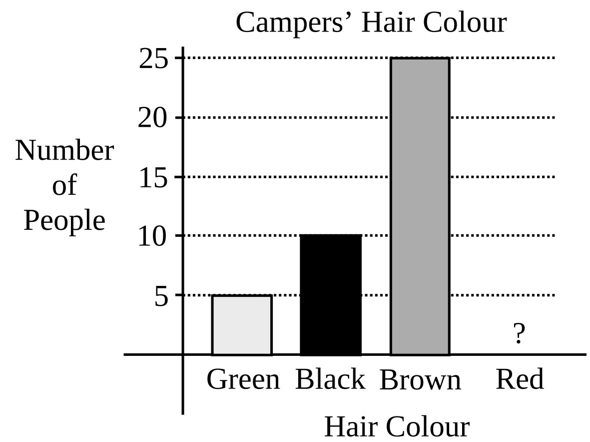
8. The area of the entire figure shown is
(**A**) 16 (**B**) 32 (**C**) 20
(**D**) 24 (**E**) 64



Solution
Each of the three small triangles is an isosceles right angled triangle having a side length of 4. The area of each small triangle is thus $\frac{1}{2}(4)(4) = 8$. The total area is 3×8 or 24. ANSWER: (D)

9. The bar graph shows the hair colours of the campers at Camp Gauss. The bar corresponding to redheads has been accidentally removed. If 50% of the campers have brown hair, how many of the campers have red hair?

(A) 5 **(B)** 10 **(C)** 25 **(D)** 50 **(E)** 60



Solution

From the graph, we can see that there are 25 campers with brown hair. We are told that this represents 50% of the total number of campers. So in total then there are 2×25 or 50 campers. There is a total of 15 campers who have either green or black hair. This means that $50-(25+15)$ or 10 campers have red hair. ANSWER: (B)

10. Henri scored a total of 20 points in his basketball team's first three games. He scored $\frac{1}{2}$ of these points in the first game and $\frac{1}{10}$ of these points in the second game. How many points did he score in the third game?

(A) 2 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 8

Solution

Henri scored $\frac{1}{\cancel{2}_{1}}\times\cancel{20}^{10}$ or 10 points in his first game. In his second game, he scored $\frac{1}{\cancel{10}_{1}}\times\cancel{20}^{2}$ or 2 points.

In the third game, this means that he will score $20-(2+10)$ or 8 points.

ANSWER: (E)

Part B

11. A fair die is constructed by labelling the faces of a wooden cube with the numbers 1, 1, 1, 2, 3, and 3. If this die is rolled once, the probability of rolling an odd number is

(A) $\frac{5}{6}$ **(B)** $\frac{4}{6}$ **(C)** $\frac{3}{6}$ **(D)** $\frac{2}{6}$ **(E)** $\frac{1}{6}$

Solution

There are six different equally likely possibilities in rolling the die. Since five of these are odd numbers, the probability of rolling an odd number is five out of six or $\frac{5}{6}$. ANSWER: (A)

12. The ratio of the number of big dogs to the number of small dogs at a pet show is 3:17. There are 80 dogs, in total, at this pet show. How many big dogs are there?

(**A**) 12 (**B**) 68 (**C**) 20 (**D**) 24 (**E**) 6

Solution

Since the ratio of the number of big dogs to small dogs is 3:17 this implies that there are 3 large dogs in each group of 20. Since there are 80 dogs, there are four groups of 20. This means that there are 3×4 or 12 large dogs. ANSWER: (A)

13. The product of two whole numbers is 24. The smallest possible sum of these two numbers is

(**A**) 9 (**B**) 10 (**C**) 11 (**D**) 14 (**E**) 25

Solution

If two whole numbers have a product of 24 then the only possibilities are 1×24, 2×12, 3×8 and 4×6. The smallest possible sum is $4+6$ or 10. ANSWER: (B)

14. In the square shown, the numbers in each row, column, and diagonal multiply to give the same result. The sum of the two missing numbers is

(**A**) 28 (**B**) 15 (**C**) 30
(**D**) 38 (**E**) 72

12	1	18
9	6	4
		3

Solution

The numbers in each row, column and diagonal multiply to give a product of $(12)(1)(18)$ or 216. We are now looking for two numbers such that $(12)(9)(\quad)=216$ and $(1)(6)(\quad)=216$. The required numbers are 2 and 36 which have a sum of 38. ANSWER: (D)

15. A prime number is called a "Superprime" if doubling it, and then subtracting 1, results in another prime number. The number of Superprimes less than 15 is

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

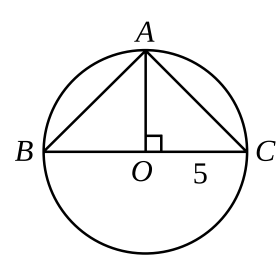
Solution

The only possible candidates for 'Superprimes' are 2, 3, 5, 7, 11 and 13 since they are the only prime numbers less than 15. If we double each of these numbers and then subtract 1 we get 3, 5, 9, 13, 21 and 25. Three of these results are prime numbers. So there are only three Superprimes.

ANSWER: (B)

16. BC is a diameter of the circle with centre O and radius 5, as shown. If A lies on the circle and AO is perpendicular to BC, the area of triangle ABC is

(**A**) 6.25 (**B**) 12.5 (**C**) 25
(**D**) 37.5 (**E**) 50



Solution
If O is the centre of the circle with radius 5 this implies that $OB = AC = OC = 5$. Thus, we are trying to find the sum of the areas of two identical isosceles right angled triangles with a side length of 5. The required area is $2\left[\frac{1}{2}(5)(5)\right]$ or 25. Answer: (C)

17. A rectangular sign that has dimensions 9 m by 16 m has a square advertisement painted on it. The border around the square is required to be at least 1.5 m wide. The area of the largest square advertisement that can be painted on the sign is
(**A**) 78 m^2 (**B**) 144 m^2 (**C**) 36 m^2 (**D**) 9 m^2 (**E**) 56.25 m^2

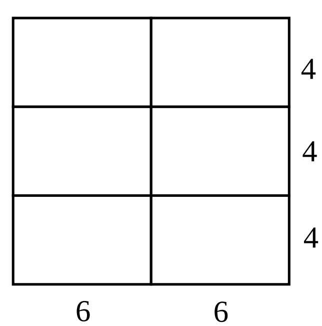
Solution
If the 9×16 rectangle has a square painted on it such that the square must have a border of width 1.5 m this means that the square has a maximum width of $9 - 1.5 - 1.5 = 6$. So the largest square has an area of $6 \text{ m} \times 6 \text{ m}$ or 36 m^2. Answer: (C)

18. Felix converted \$924.00 to francs before his trip to France. At that time, each franc was worth thirty cents. If he returned from his trip with 21 francs, how many francs did he spend?
(**A**) 3 080 (**B**) 3 101 (**C**) 256.2 (**D**) 3 059 (**E**) 298.2

Solution
If each franc has a value of \$0.30 then Felix would have been able to purchase $\frac{924}{0.30}$ or $3\,080$ francs. If he returns with 21 francs, he then must have then spent $3\,080 - 21$ or $3\,059$ francs.
Answer: (D)

19. Rectangular tiles, which measure 6 by 4, are arranged without overlapping, to create a square. The minimum number of these tiles needed to make a square is
(**A**) 8 (**B**) 24 (**C**) 4 (**D**) 12 (**E**) 6

Solution
Since the rectangles measure 6×4 this means that the lengths of their sides are in a ratio of 3:2. This implies that we need two rectangles that have 6 as their side length to form one side of the square and for each of these rectangles we need three others to form the other side length. In total, we need 2×3 or 6 rectangles.



Answer: (E)

20. Anne, Beth and Chris have 10 candies to divide amongst themselves. Anne gets at least 3 candies, while Beth and Chris each get at least 2. If Chris gets at most 3, the number of candies that Beth could get is

(**A**) 2 (**B**) 2 or 3 (**C**) 3 or 4 (**D**) 2, 3 or 5 (**E**) 2, 3, 4, or 5

Solution

If Anne gets at least 3 candies and Chris gets either 2 or 3 this implies that Beth could get as many as 5 candies if Chris gets only 2. If Chris and Anne increase their number of candies this means that Beth could get any number of candies ranging from 2 to 5. ANSWER: (E)

Part C

21. Naoki wrote nine tests, each out of 100. His average on these nine tests is 68%. If his lowest mark is omitted, what is his highest possible resulting average?

(**A**) 76.5% (**B**) 70% (**C**) 60.4% (**D**) 77% (**E**) 76%

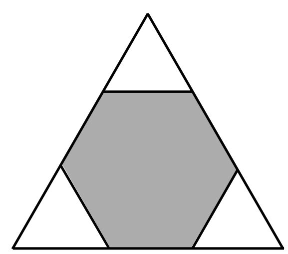
Solution

If Naoki had an average of 68% on nine tests he would have earned a total of 9×68 or 612 marks. Assuming that his lowest test was a '0', this means that Naoki would still have a total of 612 marks only this time on 8 tests. This would mean that Naoki would have an average of $\frac{612}{8}$ or 76.5%.

ANSWER: (A)

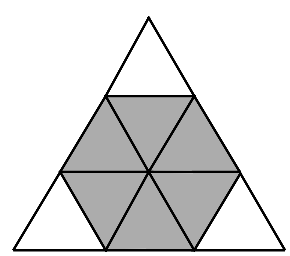
22. A regular hexagon is inscribed in an equilateral triangle, as shown. If the hexagon has an area of 12, the area of this triangle is

(**A**) 20 (**B**) 16 (**C**) 15
(**D**) 18 (**E**) 24



Solution

First of all, we can see that each of the smaller triangles is equilateral. Also, the side length of each of these is equal to that of the inscribed hexagon. From here, we can divide the hexagon up into six smaller triangles, identical to the white triangles at the three vertices as in the diagram above.
This means that each of the six triangles of the hexagon would have an area of 2, meaning that the large triangle would have an area of 9×2 or 18.



ANSWER: (D)

23. Catrina runs 100 m in 10 seconds. Sedra runs 400 m in 44 seconds. Maintaining these constant speeds, they participate in a 1 km race. How far ahead, to the nearest metre, is the winner as she crosses the finish line?

(**A**) 100 m (**B**) 110 m (**C**) 95 m (**D**) 90 m (**E**) 91 m

Solution

Since Sedra runs 400 m in 44 seconds, then she can run 100 m in 11 seconds. So Catrina runs farther than Sedra.

If Catrina runs 100 m in 10 seconds, she will complete the race in $\frac{1000}{100}\times 10$ or 100 seconds. In 100 seconds, Sedra would have run only $\frac{400}{44}\times 100$ or 909.09 metres. Sedra would then be approximately $1000-909.09$ or 90.91 metres behind as Catrina crossed the finish line. ANSWER: (E)

24. Enzo has fish in two aquariums. In one aquarium, the ratio of the number of guppies to the number of goldfish is 2:3. In the other, this ratio is 3:5. If Enzo has 20 guppies in total, the least number of goldfish that he could have is

(**A**) 29 (**B**) 30 (**C**) 31 (**D**) 32 (**E**) 33

Solution 1

The following tables give the possible numbers of fish in each aquarium.

The three lines join the results which give a total of 20 guppies, namely $2+18$, $8+12$ and $14+6$. The corresponding numbers of goldfish are 33, 32 and 31. The least number of goldfish that he could have is 31.

1st aquarium	
Number of guppies	Number of goldfish
2	3
4	6
6	9
8	12
10	15
12	18
14	21
16	24
18	27

2nd aquarium	
Number of guppies	Number of goldfish
3	5
6	10
9	15
12	20
15	25
18	30

Solution 2

In the first aquarium, the ratio of the number of guppies to goldfish is 2:3 so let the actual number of guppies be $2a$ and the actual number of goldfish be $3a$. In the second aquarium, if we apply the same reasoning, the actual number of guppies is $3b$ and the actual number of goldfish is $5b$. In total, there are 20 guppies so we now have the equation, $2a+3b=20$. We consider the different possibilities for a and b also calculating $3a+5b$, the number of goldfish. From the chart, we see that the smallest possible number of goldfish is 31.

$2a+3b$	a	b	$3a+5b$
20	1	6	33
20	4	4	32
20	7	2	31

ANSWER: (C)

25. A triangle can be formed having side lengths 4, 5 and 8. It is impossible, however, to construct a triangle with side lengths 4, 5 and 9. Ron has eight sticks, each having an integer length. He observes that he cannot form a triangle using any three of these sticks as side lengths. The shortest possible length of the longest of the eight sticks is

(**A**) 20 (**B**) 21 (**C**) 22 (**D**) 23 (**E**) 24

Solution

If Ron wants the three smallest possible lengths with which he cannot form a triangle, he should start with the lengths 1, 1 and 2. (These are the first three Fibonacci numbers). If he forms a sequence by adding the last two numbers in the sequence to form the next term, he would generate the sequence: 1, 1, 2, 3, 5, 8, 13, 21. Notice that if we take any three lengths in this sequence, we can never form a triangle. The shortest possible length of the longest stick is 21. ANSWER: (B)

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